Performance Analysis of Rate-Based Flow Control under a Variable Number of Sources

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Abstract- Rate-based Flow control plays an important role for efficient traffic management of ABR service in ATM networks. In this paper, a performance analysis of a rate-based flow control mechanism is presented. In our analytical model, the number of active sources is variable. A new source arrives when a connection is established, and an existing source departs when it has transmitted its data. Hence our model not only reflect the real scenes, but also correctly estimate the effect of the rate-based flow control.

Due to this variation, the analysis of the steady state is not enough. Therefore the analysis of transient cycles is also developed. Using the results of both analyses, we derive the equations of cell loss probability and utilization. Numerical results, confirmed by simulation, are given to show that our analysis is accurate. Also an underestimation of over an order of magnitude in cell loss probability is observed when we use the analysis ignoring the transient cycles. We demonstrate that the efficiency of using rate-based control is seriously influenced by the source arrival rate and source departure rate.

Keywords: ATM Networks; rate-based flow control; ABR(available bit rate); EFCI(explicit forward congestion indication); ER(explicit rate).

1 Introduction

ATM (Asynchronous transfer mode) is the most promising transfer technology for implementing B-ISDN. It supports applications with distinct QoS requirements such as delay, jitter, and cell loss and with distinct demands such as bandwidth and throughput. To provide these services for a wide variety of applications, in addition to CBR (Constant bit rate), rt-VBR (Real-time variable bit rate), nrt-VBR (Non-Real-Time VBR), and UBR (Unspecified bit Rate) services, the ATM forum defined a new service class known as ABR (Available bit rate) service to support data applications economically. Also an end-to-end adaptive control mechanism called closed-loop rate-based flow control is applied to this service. In this control scheme, the allowed cell transmission rate of each ABR connection is dynamically regulated by feedback information from the network [1-3]. If the network is congested, the source end decreases its cell transmission rate when it receives the congestion indication. Also, the source end increases its cell transmission rate when congestion is relieved. The rate-based control mechanism could efficiently control the connection flows and utilize the network bandwidth.

Recently several analyses and simulations have been conducted for rate-based control schemes. First Bolot and Shankar used differential equations to model the rate increase and decrease [4]. Yin and Hluchyj proposed analytical models for early versions of ABR control with a timer-based approach [5,6]. Ramamurthy and Ren developed a detailed analytical model to capture the behavior of a rate-based control scheme and obtain approximate solutions in closed forms [7]. Ohsaki et al. made an analysis and comparison between different switches in the steady state and initial transient state [8,9,10]. Ritter derived the closed form expression to quickly estimate the buffer requirements of different switches [11]. We derived the equations of cell loss probability and utilization for the rate-based control, and provided some rules to reduce cell loss probability and raise utilization [12]. These papers provide much insight into the effect of using the rate-based flow control. However, none of these papers consider the variable number of sources.

These researches under the assumption of a fixed number of sources are inaccurate. First, in real conditions, the number of sources is variable; a new connection may be established and an existing connection may be released. Second, using the rate-based control, the most obvious oscillations in buffer size happen at the time of a new source arrival or an existing source departure. Ignoring the fact of the variation about the number of sources does not correctly show the impact of rate-based control.

In this paper, we assume the number of sources to be variable to reflect the real conditions. An existing source may depart and a new source may arrive. We analyze the rate-based flow control under a variable number of sources, and deduce the equations of cell loss probability and utilization. In order to get the precise values of these parameters, besides the analysis for the steady state, the analysis for a transient state is developed. This is because the dynamic behavior is very different from the stable one when an existing source departs or a new source arrives.

The rest of this paper is organized as follows. In section 2, the rate-based control mechanism and our model are described. We analyze the rate-based flow control during the steady state in section 3. The analysis during transient cycles is presented in section 4. Using the probability and queueing theory, we derive the equations of cell loss probability and utilization in section 5. In section 6, the numerical results are obtained from our analysis and simulation. The conclusion is presented in section 7.

2 Analytical Model

First we briefly introduce the basic operation of a close-loop rate-based control mechanism [3]. When a connection is established, the source end system (SES) sends the cells at the allowed cell rate (ACR) which is set as initial cell rate (ICR). In order to probe the congestion status of the network, The SES sends a forward Resource Management (RM) cell every N_{RM} data cells. The destination end system (DES) returns the forward RM cell as a backward RM cell to the SES. Depending on the received backward RM cell, the SES adjusts its allowed cell rate, which is bounded between Peak cell rate (PCR) and Minimum cell rate (MCR).

The RM cell contains a 1-bit congestion indication (CI) which is set to zero, and an explicit rate (ER) field which is set to PCR initially by the SES. Depending on the different ways to indicate the congestion status, two types of switches are implemented. One is the Explicit Forward Congestion Indication (EFCI) switch, the other is the Explicit Rate (ER) switch. In the EFCI type, the switch in the congestion status sets the EFCI bit to one (EFCI=1) in the header of each passing data cells. The DES, if a cell with EFCI=1 has been received, marks the CI bit (CI=1) to indicate congestion in each backward RM cells. In the ER type, the switch sets the EFCI bit of the RM cells to indicate whether there is congestion or not, and sets the ER field to indicate the bandwidth the connection should use. The performance results and comparisons between the two types of switches are shown in [10] in detail.

When the SES receives a backward RM cell, it modifies its ACR using additive increase and multiplicative decrease. Depending on CI, the new ACR is computed as follows:

$$ACR = \max(\min(ACR + N_{RM} \cdot AIR, ER), MCR), \quad \text{if CI=0},$$

$$ACR = \max(\min(ACR \cdot (1 - \frac{N_{RM}}{RDF}), ER), MCR), \quad \text{if CI=1},$$

where AIR is the additive increase rate and RDF is the rate decrease factor. AIR and RDF are defined in the traffic management specification version 4.0 [3]. Although the new version of the specification has made some changes about AIR and RDF [13], the analysis for the new notations is easily translated from this paper.

In this paper, we focus on the EFCI switch and use a simple model as shown in Fig. 1. There are some ABR sources sharing a bottleneck link where the bandwidth is BW. We assume that these sources are homogeneous; that is, they all have the same parameters ICR, PCR, MCR, AIR and RDF. The number of sources is variable. Source arrival is according to a Poisson process and the duration of a source is according to an exponential process. Let λ be the source arrival rate and $1/\mu$ be the mean source duration. Thus, the distribution of the number of sources is the same as the number in the system for an M/M/∞ queue. Also we assume that each source always has cells to send, i.e. it has infinite backlog. This assumption allows us to investigate the performance of an EFCI switch in the most stressful situations.

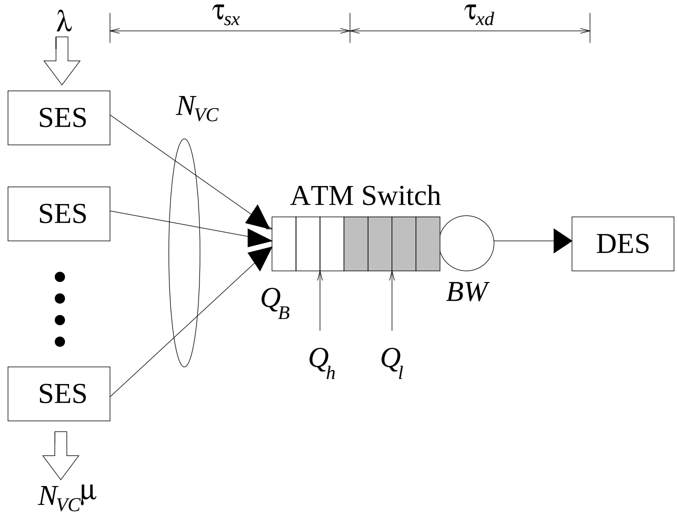


Figure 1: Analytic model for the rate-based flow control.

The buffer size at the switch is denoted by Q_B. The switch determines the congestion condition according to its queue length. There are two values, high threshold Q_h and low threshold Q_l, which decide whether congestion occurs or not. When the queue length exceeds Q_h, the EFCI bit of passing data cells is set to one to indicate congestion. The congestion is relieved when the queue length drops below Q_l.

We define τ_{sx} as the propagation delay between the SES and the switch, and τ_{xd} as the propagation delay between the switch and the DES. Also, the feedback propagation delay from the switch to the SES is denoted by τ_{xds} and the round trip propagation delay is denoted by τ. Thus we get the relation $\tau_{xds} = \tau_{sx} + 2\tau_{xd}$ and $\tau = 2(\tau_{sx} + \tau_{xd})$. The propagation delay is a critical parameter of system performance.

3 Steady State

Consider the case that the number of sources, N_{VC}, is fixed, that is, no existing source departs and no new source arrives. When a long time escapes, the system reaches the steady state which is a consequence of the same cycles. We shall derive cell loss probability and utilization in this case. First we describe the four elementary phases which composing a cycle.

3.1 Elementary phases

The evolution of the ACR can be characterized into four elementary phases differentiated by (1) the increase or decrease of the ACR, and (2) the speed of the increase or decrease. The speed of adjustment is determined by the rate RM cells are received in the SES. Meanwhile, this rate is influenced by the status of the buffer τ_{xds} time units before. So actually the evolution of the ACR is partitioned into four phases depending on the $ACR(t)$ and $Q(t-\tau_{xds})$.

- *Phase I*: $ACR(t)\uparrow$, $Q(t-\tau_{xds})>0$

During this period, each SES receives the backward RM cells with CI=0 at constant rate $BW/N_{VC}N_{RM}$. When a backward RM cell is received, the SES increases its ACR by $N_{RM}AIR$, to a rate not greater than PCR. We use a continuous time approximation to model this discrete increase for simplicity. This approximation is accurate to characterize the dynamic behavior of rate-based control [11]. Thus the following differential equation for $ACR_I(t)$ is given by

$$\frac{dACR_I(t)}{dt}=\frac{BW\cdot AIR}{N_{VC}},$$

which gives

$$ACR_I(t)=\min(ACR_I(0)+\frac{BW\cdot AIR}{N_{VC}}t, PCR). \quad (1)$$

- *Phase II*: $ACR(t)\downarrow$, $Q(t-\tau_{xds})<Q_B$

During this period, each SES receives the backward RM cells with CI=1 at constant rate $BW/N_{VC}N_{RM}$. When a backward RM cell is received, the SES reduces its ACR by $N_{RM}ACR/RDF$, to a rate not lower than MCR. We thus have the following continuous approximation for $ACR_{II}(t)$.

$$\frac{dACR_{II}(t)}{dt}=-ACR_{II}(t)\frac{BW}{N_{VC}RDF},$$

which gives

$$ACR_{II}(t)=\max(ACR_{II}(0)e^{-\frac{BW}{N_{VC}RDF}t}, MCR). \quad (2)$$

- *Phase III*: $ACR(t)\uparrow$, $Q(t-\tau_{xds})=0$

When the buffer is empty, the switch is not fully utilized. In order not to waste the bandwidth with idle time, we ought to avoid the occurrence of this phase. In this phase, RM cells arrive at the SES depending on its own rate τ time before. i.e., $ACR_{III}(t-\tau)$. So the differential equation for $ACR_{III}(t)$ is given by

$$\frac{dACR_{III}(t)}{dt} = ACR_{III}(t-\tau)\cdot AIR,$$

which gives

$$ACR_{III}(t) = \min(ACR_{III}(0)e^{\beta t}, PCR), \tag{3}$$

where β is given as the root of the equation $\beta = AIRe^{-\beta\tau}$.

- *Phase IV*: $ACR(t)\downarrow$, $Q(t-\tau_{xds})=Q_B$

When the buffer is full, the arrival cells are discarded at the switch. In order not to waste the bandwidth with retransmission of the lost cells, we also hope not to enter of this phase. In this phase, although the cell loss happens, the number of data cells between two consecutive RM cells can be approximated as N_{RM} because data cells and RM cells are both discarded in the switch when buffer is full. The approximation causes the easier analysis, and it is verified as a good approach [12]. Thus the arriving rate of RM cells at the SES is $BW/N_{VC}N_{RM}$. The behavior of ACR during this phase is given by

$$\frac{dACR_{IV}(t)}{dt} = -ACR_{IV}(t)\frac{BW}{N_{VC}RDF},$$

which gives

$$ACR_{IV}(t) = \max(ACR_{IV}(0)e^{-\frac{BW}{N_{VC}RDF}t}, MCR). \tag{4}$$

Although the equations of phase II and phase IV are the same, we differentiate between them since there is different meaning.

Note the MCR and PCR are not considered in the analysis below since we assume $MCR=0$ and $PCR=BW$. The analysis of $MCR>0$ or $PCR<BW$ can be expanded from our analysis, but the equations are more complex.

3.2 A cycle

In a cycle, phase I and phase II are the essential phases. On the other hand, phase III and phase IV may or may not occur. A cycle behaves as in Fig. 2.

When the queue length is below the low threshold, i.e., at the time t_{Q_l}, the congestion is relieved in the switch and this congestion relief signal will arrive at the SES after the propagation delay τ_{xds}. Then the SES increase its ACR and phase I begins. For convenience in explaining the time shift between $ACR(t)$ and $Q(t)$ caused by propagation delay. we use t^+ and t^- to denote the time instants "after τ_{xds}" and "before τ_{sx}" from the time t, respectively. So the phase I begins at $t^+_{Q_l}$. Similarly the beginning time of phase II is $t^+_{Q_h}$, where t_{Q_h} is the time that queue length exceeds Q_h.

Let t_0 and t_1 be two time stamps for our analysis. At the time t_0 and t_1, ACR is equal to BW/N_{VC}.

If phase IV or phase III occur, phase IV interrupts phase II and phase III interrupts phase I. When the queue length exceeds Q_B, cells loss starts. Phase IV begins at the $t^+_{Q_B}$, and consequently, the ending time of phase IV is after τ from t_0. That is because the aggregate cell arrival rate at the switch is below the bandwidth BW at time t_0, and the queue length starts to decrease after the propagation delay between the SES and the switch, τ_{sx}. Similarly, the beginning time of phase III is $t^+_{Q_0}$, where t_{Q_0} is the time that queue length reaches zero, and the ending time of phase III is τ time units after the time t_1.

3.3 Cell loss probability and utilization

Depending on which phase the time marking is located in, different cases should be considered. For example, $t^-_{Q_0}$ can be in phase I or phase II, and $t^-_{Q_h}$ can be in phase III or phase I. However, when we carefully set the Q_h and Q_l according to the rules described in the paper [12], the time between t_{Q_l} and t_{Q_0} is long and the duration of phase III is short. Therefore most situations shall be the case shown as Fig. 2 when phase III and IV appear.

First we derive the maximum queue length, Q_{max}, and minimum queue length, Q_{min}. Note that Q_{max} and Q_{min} is not limited, i.e., the value of Q_{max} might be bigger than the buffer size and the value of Q_{min} might be less than zero.

At time t_0, the aggregate cell arrival rate at the switch is below bandwidth, BW, and queue starts to decrease after the propagation delay between the SES and the switch, τ_{sx}. After the time Δt_{Q_l}, the buffer content reaches the low threshold, Q_l, and the ACR at the SES stops decreasing at $t^+_{Q_l}$. Although this period crosses phase II and phase IV, the same dynamic behavior exists for

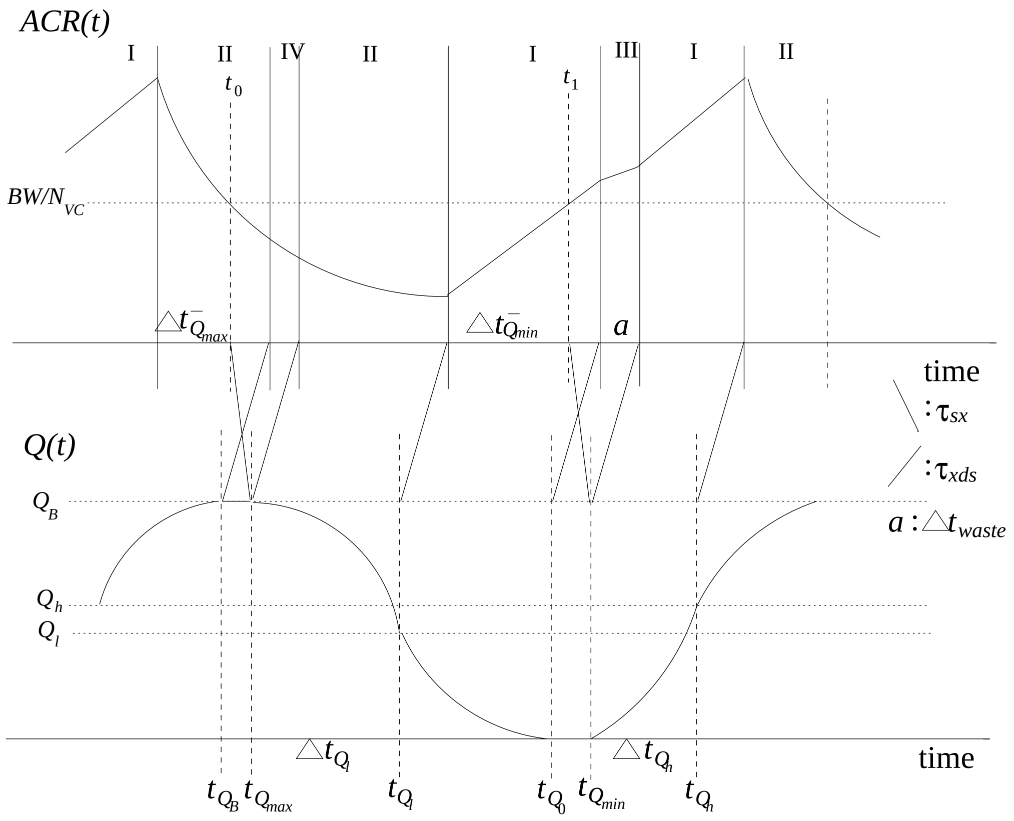


Figure 2: The evolution of ACR and Q.

phase II and phase IV. Thus we have

$$\int_0^{\Delta t_{Q_l}} BW(1 - e^{-\frac{BW}{N_{VC}\cdot RDF}t})dt = \min(Q_{max}, Q_B) - Q_l.$$

Hence the time interval Δt_{Q_l} is got as the root of

$$BW \cdot \Delta t_{Q_l} + N_{VC} \cdot RDF \cdot e^{-\frac{BW}{N_{VC}\cdot RDF}\Delta t_{Q_l}} = \min(Q_{max}, Q_B) - Q_l + N_{VC} \cdot RDF.$$

The minimum rate, ACR_{min}, is given by

$$ACR_{min} = \frac{BW}{N_{VC}} e^{-\frac{BW}{N_{VC}\cdot RDF}(\Delta t_{Q_l}+\tau)}.$$

Let $ACR_{t_0}(t)$ be the evolution of the ACR from time t_0, which is given by

$$ACR_{t_0}(t) = \frac{BW}{N_{VC}} e^{-\frac{BW}{N_{VC}\cdot RDF}t}.$$

At the time that the ACR becomes minimum, the SES begins to increase its ACR and the system enters phase I. After the time $\Delta t^-_{Q_{min}}$, the aggregate cell arrival at the switch is over BW. Hence we get

$$\Delta t^{-}_{Q_{min}} = \frac{BW - N_{VC} \cdot ACR_{min}}{BW \cdot AIR}.$$

Now the minimum queue length, Q_{min}, is given by

$$Q_{min} = \min(Q_{max}, Q_B) - \int_0^{\Delta t_{Q_l}+\tau} (BW - N_{VC} \cdot ACR_{t_0}(t))dt - \int_0^{\Delta t^{-}_{Q_{min}}} (BW - N_{VC} \cdot ACR_I(t))dt.$$

Using $ACR_I(0) = ACR_{min}$, Δt_{Q_l}, $\Delta t^{-}_{Q_{min}}$, and $ACR_{t_0}(t)$ from the above equations, finally we have

$$Q_{min} = Q_l - \tau \cdot BW + N_{VC} \cdot RDF \cdot e^{-\frac{BW}{N_{VC} \cdot RDF} \cdot \Delta t_{Q_l}} \cdot (1 - e^{-\frac{BW}{N_{VC} \cdot RDF} \cdot \tau})$$
$$- \frac{(BW - N_{VC} \cdot ACR_{min})^2}{2 \cdot BW \cdot AIR}. \quad (5)$$

If Q_{min} is larger than zero, phase III does not occur, and no bandwidth is wasted. On the other hand, if Q_{min} is less than zero, phase III occurs. Hence the wasted bandwidth, N_{waste}, is given by

$$N_{waste} = \max(0, 0 - Q_{min}). \quad (6)$$

Below we consider two cases according to whether phase III occurs or not.

• **Case 1** : phase III does not occur

First we get the equation of the interval, Δt_{Q_h}, which is the time that the queue length increases from Q_{min} to Q_h. Since the increasing rate of ACR at each SES is $BW \cdot AIR/N_{VC}$ from equation 1, we get

$$\Delta t_{Q_h} = \sqrt{\frac{2(Q_h - Q_{min})}{BW \cdot AIR}}.$$

The maximum rate, ACR_{max}, is given by

$$ACR_{max} = \frac{BW}{N_{VC}}(1 + AIR \cdot (\tau + \Delta t_{Q_h})).$$

At the time that the ACR becomes maximum, the SES begins to decrease its ACR and the system enters phase II. After the time $\Delta t^{-}_{Q_{max}}$, the aggregate cell arrival at the switch is below BW, and the queue length begins to decrease. So we have

$$ACR_{max} e^{-\frac{BW}{N_{VC} RDF} \Delta t^{-}_{Q_{max}}} = BW/N_{VC},$$

which is solved as

$$\Delta t^{-}_{Q_{max}} = -\frac{N_{VC} \cdot RDF}{BW} log \frac{BW}{N_{VC} \cdot ACR_{max}}.$$

Now the maximum queue length, Q_{max}, is given by

$$Q_{max} = Q_{min} + \int_0^{\Delta t_{Q_h}+\tau} (BW \cdot AIR \cdot t)dt + \int_0^{\Delta t^-_{Q_{max}}} (N_{VC} \cdot ACR_{II}(t) - BW)dt.$$

Using $ACR_{II}(0) = ACR_{max}$, Δt_{Q_h}, $\Delta t^-_{Q_{max}}$ and $ACR_{II}(t)$ from above equation, finally we have

$$Q_{max} = Q_h + \tau \cdot \sqrt{2(Q_h - Q_{min}) \cdot BW \cdot AIR} + \frac{\tau^2 \cdot BW \cdot AIR}{2} + N_{VC} \cdot RDF(log(\frac{BW}{N_{VC} \cdot ACR_{max}}) + \frac{N_{VC} \cdot ACR_{max}}{BW} - 1). \quad (7)$$

- **Case 2:** phase III occurs

In order to get the ACR_{max} in this case, first we compute the length of phase III, Δt_{waste}, as

$$\Delta t_{waste} = \Delta t^-_{Q_{min}} - \Delta t^-_{Q_0},$$

where $\Delta t^-_{Q_0}$ is the interval between the time of the minimum rate and the time $t^-_{Q_0}$. We get $\Delta t^-_{Q_0}$ by solving the equation,

$$0 = \min(Q_B, Q_{max}) - \int_0^{\Delta t_{Q_l}+\tau} (BW - N_{VC} \cdot ACR_{t_0}(t))dt - \int_0^{\Delta t^-_{Q_0}} (BW - N_{VC} \cdot ACR_I(t))dt.$$

Now we calculate the ACR_{max}:

$$ACR_{III}(0) = \frac{BW}{N_{VC}} + \frac{BW \cdot AIR}{N_{VC}}(\tau - \Delta t_{waste}), \quad \text{in phase I,}$$

$$ACR_I(0) = ACR_{III}(0)e^{\beta \Delta t_{waste}}, \quad \text{in phase III,}$$

$$ACR^-_{Q_h} = ACR_I(0) + \frac{BW \cdot AIR}{N_{VC}} \Delta t^-_{Q_h}, \quad \text{in phase I,}$$

$$ACR_{max} = ACR^-_{t_{Q_h}} + \frac{BW \cdot AIR}{N_{VC}} \tau, \quad \text{in phase I,}$$

where $\Delta t^-_{Q_h}$ is the interval between the beginning time of phase I and the time $t^-_{Q_h}$. We get it by solving the equation,

$$Q_h = \int_0^{\tau - \Delta t_{waste}} BW \cdot AIR \cdot tdt + \int_0^{\Delta t_{waste}} (N_{VC} ACR_{III}(0)e^{\beta t} - BW)dt + \int_0^{\Delta t^-_{Q_h}} (N_{VC}(ACR_I(0) + \frac{BW \cdot AIR}{N_{VC}} t) - BW))dt.$$

The maximum queue length is obtained as

$$Q_{max} = Q_h + \int_0^{\tau} (N_{VC}(ACR^-_{Q_h} + \frac{BW \cdot AIR}{N_{VC}} t) - BW)dt + \int_0^{\Delta t^-_{Q_{max}}} (N_{VC} \cdot ACR_{II}(t) - BW)dt. \quad (8)$$

After we get the maximum queue length according to equation 7 or 8, the number of lost cells in a cycle, N_{loss}, is given by

$$N_{loss} = \max(0, Q_{max} - Q_B). \tag{9}$$

Now we get the cycle time T_{cycle} as

$$T_{cycle} = 2\tau + \Delta t_{Q_l} + \Delta t^{-}_{Q_{min}} + \Delta t_{Q_h} + \Delta t^{-}_{Q_{max}}. \tag{10}$$

Finally cell loss probability and utilization are as follows:

$$\bar{\rho} = 1 - \frac{N_{waste}}{T_{cycle} \cdot BW},$$

$$\bar{P}_{loss} = \frac{N_{loss}}{T_{cycle} \cdot \bar{\rho} \cdot BW + N_{loss}}. \tag{11}$$

For each term A appearing in our previous equations, let $A[i]$ be the value of A when N_{VC} is equal to i. For example, $\bar{P}_{loss}[3]$ is cell loss probability in the steady state when three sources are active.

4 Transient Cycles

Now consider the case that the number of sources is variable, i.e., an existing source may depart and a new source may arrive. Hence, besides the steady state of a fixed number of active sources, we are also concerned with cell loss probability and utilization during transient cycles, which differ considerably from the analysis of the steady state.

As we know, the cell loss probability and utilization during a transient cycle are determined by the time instant when the number of sources is changed. The worst case to cell loss probability happens as a new source arrives at the time instant when the high threshold is reached, $t^{-}_{Q_h}$. The reason is as follows. If a new source arrives after $t^{-}_{Q_h}$, this new source does not send its cells during the time interval between congestion detection and this new source arrival. Hence, cell loss probability is smaller. On the other hand, the congestion is detected earlier if a new source arrives before $t^{-}_{Q_h}$. Therefore, the maximum ACR is smaller, which causes cell loss probability to be smaller [11]. Thus, the closer the new source arrival to $t^{-}_{Q_h}$, the higher cell loss probability we got. Similarly the worst case of utilization is that an existing source departs at the time instant when the low threshold is attained, $t^{-}_{Q_l}$. In this section, we only consider these worst cases. Therefore, we shall obtain the upper bound of cell loss probability and lower bound of utilization during transient cycles.

4.1 Source departure

Without loss of generality, we assume that the number of active sources is i. When an existing source departs at the time instant $t^-_{Q_l}$, the speed of adjusting ACR is not changed immediately. After the switch has sent all the cells in the buffer, then the SES speeds up its adjustment of the ACR. That is, the rate of SES receiving the backward RM cells is changed from BW/iN_{RM} to $BW/(i-1)N_{RM}$ after the time $Q_l/BW+\tau$. Now we derive utilization at worst case.

The minimum rate, ACR_{min}, in a transient cycle is the same as in the steady state. Thus we get the time interval, $\Delta\tilde{t}^-_{Q_{min}}[i\text{-}1]$, during a transient cycle where the number of active sources changes from i to i-1, by solving the equation,

$$(ACR_{min}[i]+\frac{BW\cdot AIR}{i}\cdot\frac{Q_l}{BW})(i-1)+BW\cdot AIR\cdot(\Delta\tilde{t}^-_{Q_{min}}[i-1]-\frac{Q_l}{BW})=BW.$$

The minimum queue length, $\tilde{Q}_{min}[i\text{-}1]$, during a transient cycle where the number of active sources changes from i to i-1, is given by

$$\tilde{Q}_{min}[i-1]=Q_l-\int_{\Delta t_{Q_l}}^{\Delta t_{Q_l}+\tau}(BW-(i-1)\cdot ACR_{t_0}(t))dt$$

$$-\int_0^{\frac{Q_l}{BW}}(BW-(i-1)(ACR_{min}[i]+\frac{BW\cdot AIR}{i}t))dt$$

$$-\int_0^{(\Delta\tilde{t}^-_{Q_{min}}[i-1]-\frac{Q_l}{BW})}(BW-(i-1)(ACR_{min}[i]+\frac{Q_lAIR}{i})-BW\cdot AIR\cdot t)dt.$$

Then we get

$$\tilde{Q}_{min}[i-1]=Q_l-\tau\cdot BW+(i-1)\cdot RDF\cdot e^{-\frac{BW}{i\cdot RDF}\cdot\Delta t_{Q_l}}\cdot(1-e^{-\frac{BW}{i\cdot RDF}\cdot\tau})$$

$$-\frac{(2BW-(i-1)(2ACR_{min}[i]+\frac{Q_lAIR}{i}))\frac{Q_l}{BW}}{2}-\frac{(BW-(i-1)(ACR_{min}[i]+\frac{Q_lAIR}{i}))^2}{2\cdot BW\cdot AIR}. \quad (12)$$

After the queue length reaches the minimum, the system enters the steady state where N_{VC} is equal to i-1. Hence cell loss probability and utilization during this transient cycle are calculated as

$$\tilde{\rho}_1[i-1]=1-\frac{\tilde{N}_{waste1}[i-1]}{\tilde{T}_{cycle1}[i-1]\cdot BW},\quad\text{and}$$

$$\tilde{P}_{loss1}[i-1] = \frac{N_{loss}[i-1]}{\tilde{T}_{cycle1}[i-1] \cdot \tilde{\rho}_1[i-1] \cdot BW + N_{loss}[i-1]}. \tag{13}$$

where

$$\tilde{N}_{waste1}[i-1] = \max(0, 0 - \tilde{Q}_{min}[i-1]),$$

$$\tilde{T}_{cycle1}[i-1] = 2\tau + \Delta t_{Q_l}[i-1] + \Delta \tilde{t}^{-}_{Q_{min}}[i-1] + \Delta t_{Q_h}[i-1] + \Delta t^{-}_{Q_{max}}[i-1].$$

4.2 Source arrival

In the steady state or the transient cycle of a source departure, because the ACR of each source is the same, the speed of adjusting the ACR is the same among all sources. Therefore, we do not need to take care the scenarios that different sources bring. In contrast, in the transient phase of a new source arrival, the ACR of the new source is different from the existing sources. Then, for this new source, the speed of adjusting the ACR is also different from the existing sources. It depends on a '*rate ratio*' a queueing delay before, i.e., right before the RM cell joins the switch queue. The rate ratio, R, is the ratio between the ACR of the new source and the ACR of the existing sources.

Since the rate ratio and queueing delay are changed continuously, analyzing the speed of adjusting the ACR during this transient cycle is very difficult. We introduce the concept of '*average interval*' to alleviate this difficulty. The forepart of this transient cycle is divided into some average intervals, whose lengths are not fixed, as shown in Fig 3. In the nth average interval ΔI_n, the switch sends all the cells which have been sent by the sources in the previous average interval ΔI_{n-1}. Also the variable behavior of rate ratio is approximated by a constant, which is the ratio between the average ACR of the new source and the average ACR of the existing sources during this average interval. Hence the speed of adjusting ACR in the ΔI_n completely depends on the rate ratio, R_{n-1}, in the ΔI_{n-1}. The approximation works well when the length of average intervals is not large.

When a new source arrives at the time instant $\Delta t^{-}_{Q_h}$, the SES sends its cells at the rate ICR. The new source keeps this rate until the first RM cell returns. Let the first average interval begin at the time instant $t^{-}_{Q_h}$, and end at the time instant when the first RM cell of this new source returns. Hence the length of the first average interval is

$$\Delta I_1 = \frac{Q_h}{BW} + \tau.$$

For simplicity of presentation, let $ACR''_j(t)$ and $ACR'_j(t)$ be the dynamic behavior of the new source and of those existing sources in the jth average interval, respectively. Note that t is the escaped

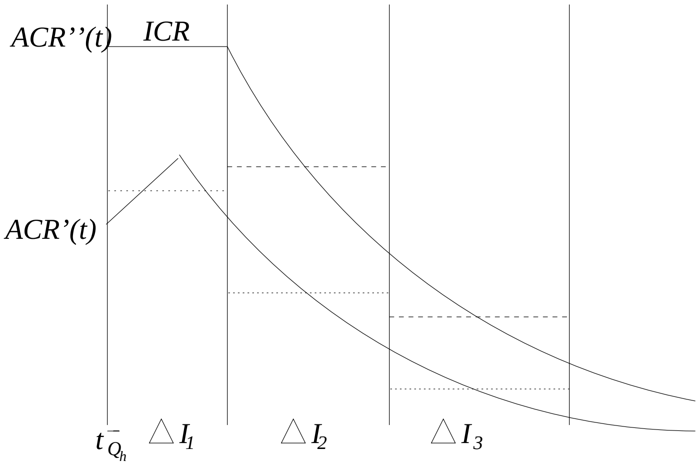


Figure 3: Transient cycle of a new source arrival.

time from the beginning time of the average interval. Now we get the dynamic behavior of the new source and the existing sources in the first average interval as

$$ACR_1''(t) = ICR, \qquad 0 \leq t < \Delta I_1,$$

$$ACR_1'(t) = \frac{BW}{i}(1 + AIR(\Delta t_{Q_h} + t)), \qquad 0 \leq t < \tau,$$

$$ACR_1'(t) = \frac{BW}{i}(1 + AIR(\Delta t_{Q_h} + \tau))e^{-\frac{BW}{iRDF}(t-\tau)}, \qquad \tau \leq t < \Delta I_1.$$

The rate ratio R_1 is given by

$$R_1 = \frac{\int_0^{\Delta I_1} ACR_1''(t)}{\int_0^{\Delta I_1} ACR_1'(t).}$$

The system is in phase II or IV after the first average interval, so the dynamic behavior is as

$$ACR_n''(t) = ACR_n''(0)e^{-\frac{BW \cdot R_{n-1}}{(i+R_{n-1})RDF}t}, \qquad 0 \leq t < \Delta I_n,$$

$$ACR_n'(t) = ACR_n'(0)e^{-\frac{BW}{(i+R_{n-1})RDF}t}, \qquad 0 \leq t < \Delta I_n.$$

where $ACR_n''(0)$ and $ACR_n'(0)$ equal to $ACR_{n-1}''(\Delta I_{n-1})$ and $ACR_{n-1}'(\Delta I_{n-1})$, respectively. The length of the second and nth average intervals is

$$\Delta I_2 = \frac{\int_0^{\Delta I_1}(iACR_1'(t) + ACR_1''(t))dt}{BW} - \tau,$$

$$\Delta I_n = \frac{\int_0^{\Delta I_{n-1}}(iACR_{n-1}'(t) + ACR_{n-1}''(t))dt}{BW}, \quad n > 2.$$

The rate ratio of the nth average interval is

$$R_n = \frac{\int_0^{\Delta I_n} ACR''_n(t)dt}{\int_0^{\Delta I_n} ACR'_n(t)dt}.$$

Without loss of generality, we assume that the queue length reaches maximum at the mth average interval.

$$iACR'_m(\tilde{t}[i+1]) + ACR''_m(\tilde{t}[i+1]) = BW.$$

Thus we get

$$\tilde{Q}_{max}[i+1] = (\Delta I_m - \tilde{t}[i+1])BW + \int_0^{\tilde{t}[i+1]} (iACR'_m(t) + ACR''_m(t))dt.$$

We assume that the system enters the steady state after the time instant when the queue length reaches the maximum. Hence cell loss probability and utilization during this transient cycle is calculated as

$$\tilde{\rho}_2[i+1] = 1 - \frac{N_{waste}[i+1]}{\tilde{T}_{cycle2}[i+1] \cdot BW},$$

$$\tilde{P}_{loss2}[i+1] = \frac{\tilde{N}_{loss2}[i+1]}{\tilde{T}_{cycle2}[i+1] \cdot \tilde{\rho}_2[i+1] \cdot BW + \tilde{N}_{loss2}[i+1]}, \tag{14}$$

where

$$\tilde{N}_{loss2}[i+1] = \max(0, \tilde{Q}_{max}[i+1] - Q_B),$$

$$\tilde{T}_{cycle2}[i+1] = 2\tau + \Delta t_{Q_h}[i+1] + \sum_{j=1}^{m-1} \Delta I_j[i+1] + \tilde{t}[i+1] + \Delta t_{Q_l}[i+1] + \Delta t_{\bar{Q}_{min}}[i+1].$$

5 Combination of Steady State and Transient Cycles

Now we derive the equations of cell loss probability and utilization. Remind that source arrival is according to a Poisson process with parameter λ, and the source existence duration is according to an exponential process with parameter $1/\mu$. The distribution of the number of sources is the same as the number in the system for an M/M/∞ queue. Hence we get

$$P_i = \frac{(\frac{\lambda}{\mu})^i}{i!} e^{-\frac{\lambda}{\mu}},$$

$$T_i = \frac{1}{\lambda + i\mu},$$

where P_i is the probability that i sources are active in the system, and T_i is the mean duration when i sources are active.

Because the event that a new source arrives or an existing source departs happen not very often, λ and μ are small. Therefore many cycles may pass by between two events. There are few opportunities that an event happens during a transient cycle. So the behavior of system looks like Fig. 4. When there are $i+1$ sources in the system and an existing source departs, the system passes the time interval of a transient cycle, and then enters the steady state. Hence the time of the system staying in the transient cycle is $\tilde{T}_{cycle1}[i]$, and the time of the system staying in the steady state is T_i-$\tilde{T}_{cycle1}[i]$. We use the probability method to obtain cell loss probability and utilization as

$$P_{loss1}[i] = \tilde{P}_{loss1}[i]\frac{\tilde{T}_{cycle1}[i]}{T_i} + \bar{P}_{loss}[i]\frac{T_i - \tilde{T}_{cycle1}[i]}{T_i},$$
$$\rho_1[i] = \tilde{\rho}_1[i]\frac{\tilde{T}_{cycle1}[i]}{T_i} + \bar{\rho}[i]\frac{T_i - \tilde{T}_{cycle1}[i]}{T_i}. \tag{15}$$

When there are $i-1$ sources in the system and a new source arrives, similarly we get $P_{loss2}[i]$ and $\rho_2[i]$.

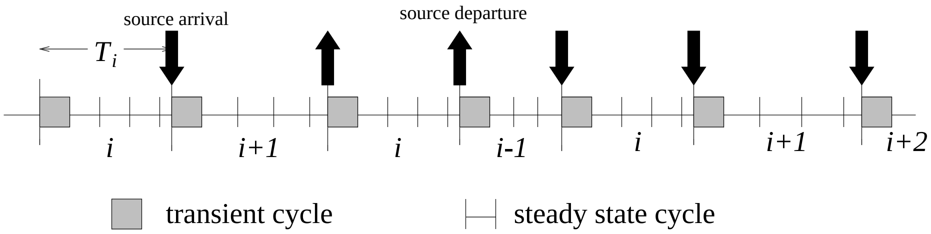


Figure 4: The evolution of the number of active sources.

As described above, there are two ways that the system enters the condition that i sources are active. The departure rate of an existing source when there are $i+1$ sources is $P_{i+1}(i+1)\mu$. On the other hand, the arrival rate of a new source when there are $i-1$ sources is $P_{i-1}\lambda$. Hence the probability of first way is $\frac{P_{i+1}(i+1)\mu}{P_{i+1}(i+1)\mu+P_{i-1}\lambda}$, and the probability of second way is $\frac{P_{i-1}\lambda}{P_{i+1}(i+1)\mu+P_{i-1}\lambda}$. Also according to the local-balance equation of the queueing theory, the rate $P_{i+1}(i+1)\mu$ is equal to $P_i\lambda$, and the rate $P_{i-1}\lambda$ is equal to $P_i i\mu$. Therefore the probabilities of first and second ways are $\frac{\lambda}{\lambda+i\mu}$ and $\frac{i\mu}{\lambda+i\mu}$, respectively.

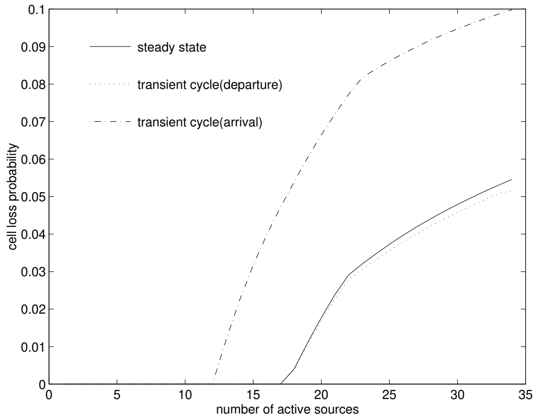
Finally we obtain cell loss probability and utilization as

$$P_{loss} = \sum_{i=0}^{\infty} P_i\left(\frac{\lambda}{\lambda+i\mu}P_{loss1}[i] + \frac{i\mu}{\lambda+i\mu}P_{loss2}[i]\right),$$
$$\rho = \sum_{i=0}^{\infty} P_i\left(\frac{\lambda}{\lambda+i\mu}\rho_1[i] + \frac{i\mu}{\lambda+i\mu}\rho_2[i]\right). \tag{16}$$

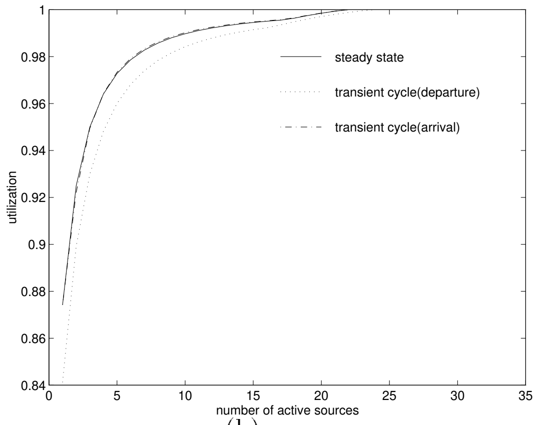
Although the summation is infinite, we have to limit the number of sources. We can set a reasonable bound for N_{VC} so that boundary states have probability very close to 0.

6 Results and Discussion

We present some numerical examples to show the correctness of our analyses on cell loss probability and utilization. The parameters of the systems are set to BW=155Mbps, PCR=155Mbps, $ICR=PCR/5$, $AIR=PCR/2048$, RDF=512, τ_{sx}:τ_{xd}=1:1, τ=1ms, N_{RM}=32, Q_B=3000, Q_h = 500, Q_l=500, λ=1 (1/sec), and μ=0.1. Thus the mean number of sources is 10.



(a)



(b)

Figure 5: Differences between the steady state and transient cycles. (a)cell loss probability (b) utilization.

Figure 5 shows cell loss probability and utilization as a function of the number of active sources. In figure 5(a), we see that cell loss probability in a transient cycle of source arrival is larger than the one in a steady state. On the other hand, cell loss probability in a transient cycle of source departure is smaller than the one in a steady state. It is very reasonable that the arrival of a source raises cell loss probability and the departure of a source reduces this probability. The similar phenomenon exists in utilization, as in fig 5(b). The arrival of a source raises utilization and the departure of a source reduces utilization.

To investigate the influence of the source arrival rate and departure rate, results for the mean number of sources equals to 10 are shown in figure 6 and 7. By changing λ and μ, we can keep the mean number of sources constant and alter the value of T_i. The range of λ is from 0.1 to 10, which corresponds to the duration 150 times to 1.5 times of cycle time approximately. From these figures, we demonstrate that the efficiency of using rate-based control is seriously influenced by the source arrival rate and the source departure rate. When the source arrival rate becomes large, cell loss probability increases. Meanwhile, when the source departure rate becomes large, utilization degrades.

Our approach is appropriate, as shown in figure 6, to estimate the real values of cell loss probability. The analytical and simulation results agree well. Since the analysis in the transient cycles considers the worst cases, the theoretical results slightly overestimate the real values. However, the distance between them shrinks when the source arrival rate becomes large. As we know, the probability, that the source arrives at a transient cycle caused by the previous source arrival, increases when the arrival rate is large. This condition causes the extra cell loss which is not considered in our analysis. The extra cell loss probability offsets the gap between our analytical and simulation values.

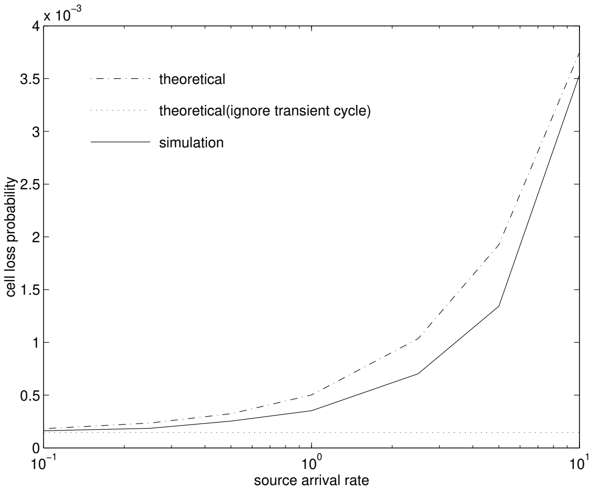


Figure 6: Effect of λ on cell loss probability (mean N_{VC}=10).

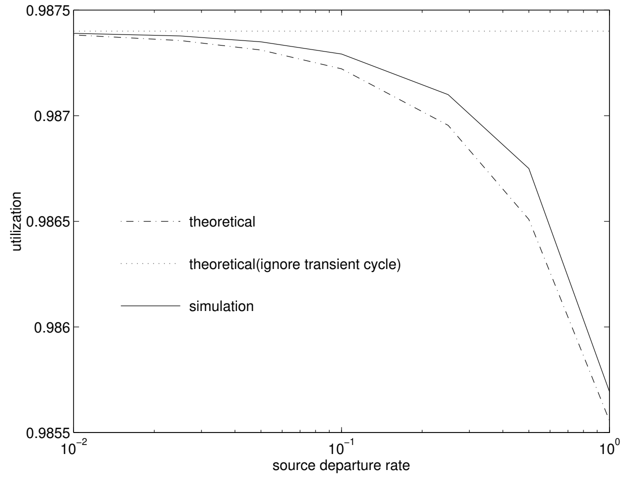


Figure 7: Effect of μ on utilization (mean N_{VC}=10).

As a comparison target, we also obtain the results which ignore the behavior of transient cycles. In this condition, cell loss probability is calculated as the summation of the product of cell loss probability under i sources in the steady state and the probability of having i sources. Due to ignoring the transient cycles, the result is always a constant regardless of the change of λ. However, it becomes less accurate as the arrival rate gets higher. When λ is 10, an accurate value is eighteen times larger than the analytical one which ignores the transient cycles.

The influence of source departure rate on utilization is shown in figure 7. The similar results are obtained as in Fig. 6, and the similar reasons can explain that.

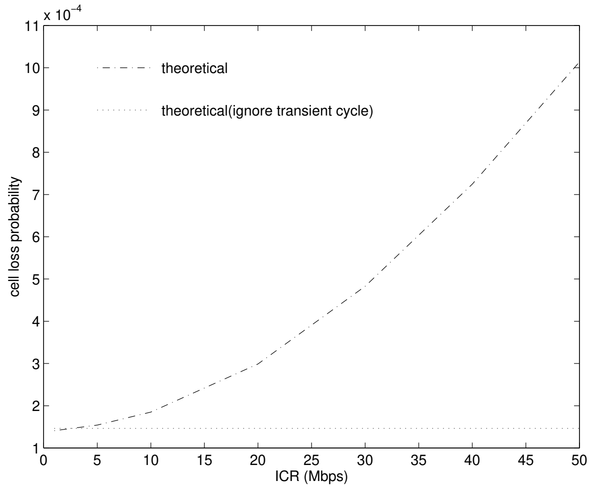


Figure 8: Effect of ICR on cell loss probability.

Figure 8 illustrates cell loss probability as a function of ICR. As we can see in the figure, cell loss probability quickly increases, almost linearly with the increase of ICR. Hence appropriate choice of ICR is necessary to avoid too much cell loss, with no sacrifice of utilization. Note when

ICR is small, our analytical results are lower than the results which ignore the transient cycles. The phenomenon is due to the larger reduction of cell loss probability in a transient cycle of source departure, although the smaller increment occurs in a transient cycle of source arrival when ICR is small.

7 Conclusion

When the variation of the number of sources is neglected, the derived cell loss probability and utilization do not correctly reflect the effect of the rate-based flow control. Meanwhile, although the variation in the number of sources is considered, the results of ignoring the transient cycles are still unsatisfactory. Simulation results show an underestimation of over an order of magnitude in cell loss probability and a little overestimation in utilization. Hence a better analysis is needed.

In this paper, an accurate analysis for rate-based flow control under a variable number of sources is provided. Numerical results, confirmed by simulation, demonstrate that our analysis is appropriate for estimating cell loss probability and utilization, no matter what values the source arrival rate and departure rate are. Also the results show the cell loss probability is obviously raised as the source arrival rate increases.

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